

D 51761

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Name.....

Reg. No.....

**THIRD SEMESTER (CBCSS—UG) DEGREE EXAMINATION  
NOVEMBER 2023**

Mathematics

MTS 3C 03—MATHEMATICS—3

(2019—2022 Admissions)

Time : Two Hours

Maximum : 60 Marks

**Part A**

*All questions can be attended.*

*Each question carries 2 marks.*

*Overall Ceiling is 20.*

1. If  $r(t) = \cos 2t i + \sin t j$ . Find  $r'(0)$ .
2. Find the curvature of a circle of radius  $a$ .
3. Describe the level surfaces of the function  $F(x, y, z) = \frac{(x^2 + y^2)}{z}$ .
4. If  $F = (x^2 y^3 - z^4) i + 4x^5 y^2 z j + y^4 z^6 k$ , find  $\text{div}(\text{curl } F)$ .
5. Evaluate  $\int xy^2 dy$  on the quarter-circle  $C$  defined by  $x = 4 \cos t, y = 4 \sin t, 0 \leq t \leq \frac{\pi}{2}$ .
6. Find  $\int_C y dx + x dy$  on the curves  $y = \sqrt{x}$  between  $(0, 0)$  and between  $(1, 1)$ .
7. Convert  $(6, \pi/4, \pi/3)$  in spherical coordinates to rectangular co-ordinates.
8. Find the values of  $\ln(-1, -i)$ .
9. Prove that  $\sinh z = \sinh x \cos y + i \cosh x \sin y$ .

**Turn over**

10. Evaluate  $\int (z + 3) dz$ , where  $C$  is  $x = 2t, y = 4t - 1, 1 \leq t \leq 3$ .
11. Evaluate  $\oint_C z^3 - 1 + 3i dz$ , where  $C$  is the circle  $|z| = 1$ .
12. State Cauchy's Integral Formula.

### Part B

*All questions can be attended.  
Each question carries 5 marks.  
Overall Ceiling is 30.*

13. Find an equation of the tangent plane to the graph of  $\frac{1}{2}x^2 + \frac{1}{2}y^2 - z = 4$  at  $(1, -1, 5)$ .
14. Find the maximum value of the directional derivative of  $F(x, y, z) = xy^2 - 4x^2y + z^2$  at  $(1, -1, 2)$  in the direction of  $6i + 2j + 3k$ .
15. Find the moment of inertia about the  $y$ -axis of the thin homogeneous disk  $x^2 + y^2 = r^2$  of mass  $m$ .  
Given  $\rho(x, y) = \frac{m}{\pi r^2}$ .
16. Find the volume of the solid that is under the hemisphere  $z = \sqrt{1 - x^2 - y^2}$  and above the region bounded by the graph of the circle  $x^2 + y^2 - y = 0$ .  $V = \iint_R \sqrt{1 - x^2 - y^2} dA$ .
17. (a) Verify that the function  $u(x, y) = x^3 - 3xy^2 - 5y$  is harmonic in the entire complex plane.  
(b) Find the harmonic conjugate function of  $u$ .
18. Solve the equation  $\cos z = 10$ .
19. Evaluate  $\oint_C \frac{dz}{z^2 + 1}$  where  $C$  is the circle  $|z| = 3$ .

**Part C**

*Answer any one question.  
The question carries 10 marks.*

20. Verify Stokes theorem. Assume that the surface  $S$  is oriented upward. Given  $F = z i + x j + y k$ ;  $S$  that portion of the plane  $2x + y + 2z = 6$  in the first octant.
21. Let  $D$  be the region bounded by the hemisphere  $x^2 + y^2 + (z - 1)^2 = 9, 1 \leq z \leq 4$ , and the plane  $z = 1$ . Verify the divergence theorem if  $F = xi + yj + (z - 1)k$ .

(1 × 10 = 10 marks)